

The Complete Bosonic Basis For A Higgs-Like Dilaton

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A Higgs-like dilaton owns couplings that differ from those of the Standard Model Higgs and of a generic Composite Higgs. The complete bosonic basis for a Higgs-like dilaton is presented at the first subleading order. The comparison with Standard Model, SMEFT and with the generic Lagrangian for the minimal $SO(5)/SO(4)$ Composite Higgs model is performed. Observables that can disentangle the different hypotheses are identified.

I. INTRODUCTION

The observation of a neutral, CP-even scalar particle at LHC [1, 2] represents a unique opportunity to investigate the scalar sector of the Standard Model (SM) and to shed light on the electroweak symmetry breaking (EWSB) mechanism. At present, there is no evidence for deviations from the SM scalar boson (“Higgs” for short) hypothesis – see Ref. [3] for a recent update. However, other theories with Higgs-like candidates are still equally viable within the present sensitivities: models with more than one Higgs doublet, such as supersymmetry or two-Higgs-doublet constructions; Composite Higgs (CH) theories; models where the Higgs arises as a Goldstone boson. Disentangling the different alternatives, considering the various experimental facilities, is crucial.

This has been deeply pursued adopting the Effective Field Theory approach, which allows to avoid the specificity of the distinct models and, instead, provides model-independent predictions characteristic of more general frameworks. Models where the physical Higgs particle belongs to a doublet representation of the electroweak (EW) symmetry can be described at low-energy by the so-called SMEFT Lagrangian [4, 5]. Otherwise, a more suitable description is the so-called HEFT Lagrangian [6–12].

The two effective Lagrangians provide a description of gauge, fermion and Higgs couplings, respecting Lorentz and $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge invariance. The key difference between the two approaches is the relationship between the physical Higgs field $h(x)$ and the SM Goldstone bosons (GBs) $\vec{\pi}(x)$: in the SMEFT, the four fields belong to the $SU(2)_L$ doublet $\Phi(x)$,

$$\Phi(x) = \mathbf{U}(x) \begin{pmatrix} 0 \\ \frac{v+h(x)}{\sqrt{2}} \end{pmatrix}, \quad (1)$$

with

$$\mathbf{U}(x) \equiv e^{i\vec{\sigma} \cdot \vec{\pi}(x)/v} \quad (2)$$

being the GB matrix. On the contrary, in the HEFT the physical Higgs and the GB matrix are treated as independent objects [6–19], which leads to a much larger number of operators in the HEFT with respect to the SMEFT, at the same order in the expansion [20–22]. Moreover, from the adimensionality of the GB matrix it follows a reshuffling of the leading operators in the HEFT with respect to the SMEFT Lagrangian. In Refs. [14–17, 23] it has been shown that HEFT exhibits the following specific features:

- some correlations typical of the SMEFT, such as those between triple and quartic gauge couplings, are lost in the HEFT;
- Higgs couplings to gauge bosons are correlated to pure gauge couplings in the SMEFT, while are completely free in the HEFT;
- a few couplings that are expected to be strongly suppressed in the SMEFT, are instead predicted with higher strength in the HEFT and potentially lead to visible observables in the present LHC run.

The HEFT Lagrangian is a very useful tool to describe an extended class of “Higgs” models: by fixing the Lagrangian parameters, it can encode SM and SMEFT, Goldstone Boson Higgs models [24–30] and dilaton-like constructions [31–37]. Therefore, HEFT can be considered the most general description of gauge, fermion and Higgs couplings, invariant under $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge symmetry.

The comparison between HEFT and SMEFT Lagrangians has undergone an intensive investigation [14–17, 23]. As well, the matching between CH models and the HEFT also received much attention: in particular, in Refs. [38, 39], considering a CH model with a symmetric coset, it has been shown the potentiality of the HEFT Lagrangian to account for Composite Higgs models as possible ultraviolet completion.

On the other hand, the link with dilaton constructions has been less studied, even if the dilaton solution to the Hierarchy problem is attractive and largely investigated [31–37]. Moreover, recent lattice simulations of strongly interacting gauge theories predict the appearance of a scalar particle that could be interpreted as a dilaton, when the conformal behaviour sets in [40–45].

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Dilaton models are based on the fact that the SM Lagrangian is approximately scale invariant, once neglecting explicit scales associated to the EWSB mechanism and the dynamical conformal breaking due to QCD. In models where the scale invariance is broken spontaneously, a GB naturally arises [46, 47] and can then be identified with the physical Higgs field: its mass is then protected by the GB shift symmetry and can only acquire (relatively small) values, close to the symmetry breaking scale.

It is the aim of the present paper to complete the comparison between the distinct Higgs setups: SM, SMEFT, CH models and dilaton theories. For definiteness, the minimal $SO(5)/SO(4)$ CH model will be considered in the following, even if the results hold also for other models, such as for the original $SU(5)/SO(5)$ model by Georgi and Kaplan [24]. As couplings with fermions follow specific assumptions on the underlying framework, the analysis will focus on physical effects in the bosonic sector only.

The structure of the paper is as follows. First, the HEFT framework is summarised in Sect. II. Then, the most generic effective Lagrangian for a Higgs-like dilaton is constructed in Sect. III. In Sect. IV, the comparison with SM, SMEFT and the minimal $SO(5)/SO(4)$ CH model is presented, including the discussion on possible discriminating signals. Concluding remarks are provided in Sect. V. In Apps. A and B, the SMEFT Lagrangian and the Lagrangian for the minimal $SO(5)/SO(4)$ CH model, respectively, are briefly summarised.

II. THE HEFT LAGRANGIAN

The building blocks used to construct the HEFT are the SM fermions and gauge bosons, together with the GB matrix $\mathbf{U}(x)$ and the physical Higgs particle $h(x)$. The GB matrix transforms as a bi-doublet under the global $SU(2)_L \times SU(2)_R$ symmetry,

$$\mathbf{U}(x) \rightarrow L \mathbf{U}(x) R^\dagger, \quad (3)$$

where L and R are the unitary transformations of $SU(2)_L$ and $SU(2)_R$, respectively. It is typically encoded into two chiral fields

$$\begin{aligned} \mathbf{T}(x) &\equiv \mathbf{U}(x) \sigma_3 \mathbf{U}^\dagger(x), & \mathbf{T}(x) &\rightarrow L \mathbf{T}(x) L^\dagger, \\ \mathbf{V}_\mu(x) &\equiv (\mathbf{D}_\mu \mathbf{U}(x)) \mathbf{U}^\dagger(x), & \mathbf{V}_\mu(x) &\rightarrow L \mathbf{V}_\mu(x) L^\dagger, \end{aligned} \quad (4)$$

with

$$\mathbf{D}_\mu \mathbf{U}(x) \equiv \partial_\mu \mathbf{U}(x) + ig \mathbf{W}_\mu(x) \mathbf{U}(x) - \frac{ig'}{2} B_\mu(x) \mathbf{U}(x) \sigma_3, \quad (5)$$

where $\mathbf{W}_\mu(x) \equiv W_\mu^a(x) \sigma_a/2$. While both fields transform in the adjoint of $SU(2)_L$, the scalar field $\mathbf{T}(x)$ is a spurion for the $SU(2)_R$ symmetry and thus, when present in an operator, allows an easy identification of its $SU(2)_R$ non-conserving nature.

The physical Higgs $h(x)$ is an isosinglet of the SM gauge symmetry and is conventionally described through adimensional generic functions $\mathcal{F}(h/v)$ [6, 48], being $v \approx 246$ GeV the EW scale. Distinct functions $\mathcal{F}(h)$ identify scalar field manifolds with different curvature [49–51], which represents an observable measurable at LHC.

The $\mathcal{F}(h/v)$ functions are commonly written as a polynomial expansion in h/v , $\mathcal{F}(h/v) = 1 + \alpha(h/v) + \beta(h/v)^2 + \dots$, where dots account for higher powers of (h/v) . In specific realisations, the scale associated to h can be distinct from v : in Composite Higgs models, for example, this scale is larger than v and is associated to the scale at which the GBs arise after the spontaneous breaking of the initial global symmetry. According to the Naive Dimensional Analysis (NDA) [20–22], which determines the suppressions of the distinct effective operators, the scale associated to h , usually denoted by f , satisfies $f \neq v$ and $\Lambda \leq 4\pi f$, being Λ the scale that fixes the validity of the theory. However, in the HEFT formalism [17, 22], factors of v/f , being f this new scale, are accounted for in the free coefficients of the effective Lagrangian, α, β, \dots , leaving v as the only sensitive scale in the $\mathcal{F}(h/v)$ functions. In what follows the notation will be simplified, suppressing the explicit dependence on v .

Finally, SM fermions are arranged in doublets of the global $SU(2)_L$ or $SU(2)_R$ symmetries: in particular, the right-handed fields are collected in the following spinors,

$$Q_R = \begin{pmatrix} u_R \\ d_R \end{pmatrix}, \quad L_R = \begin{pmatrix} N_R \\ e_R \end{pmatrix}, \quad (6)$$

where N_R are three right-handed neutrinos, introduced here to complete the $SU(2)_R$ doublet¹.

Following Ref. [17], the HEFT Lagrangian can be written as a sum of two terms,

$$\mathcal{L}_{\text{HEFT}} \equiv \mathcal{L}_h^{(0)} + \Delta \mathcal{L}_h, \quad (7)$$

where the first one reads:

$$\begin{aligned} \mathcal{L}_h^{(0)} = & -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W_{\mu\nu}^a W^{a\mu\nu} - \frac{1}{4} \mathcal{G}_{\mu\nu}^\alpha \mathcal{G}^{\alpha\mu\nu} + \\ & + \frac{1}{2} \partial_\mu h \partial^\mu h - \frac{v^2}{4} \text{Tr}(\mathbf{V}_\mu \mathbf{V}^\mu) \mathcal{F}_C(h) - V(h) + \\ & + i \bar{Q}_L \not{D} Q_L + i \bar{Q}_R \not{D} Q_R + i \bar{L}_L \not{D} L_L + i \bar{L}_R \not{D} L_R + \\ & - \frac{v}{\sqrt{2}} (\bar{Q}_L \mathbf{U} \mathcal{Y}_Q(h) Q_R + \text{h.c.}) + \\ & - \frac{v}{\sqrt{2}} (\bar{L}_L \mathbf{U} \mathcal{Y}_L(h) L_R + \text{h.c.}) + \\ & - \frac{g_s^2}{(4\pi)^2} \lambda_s \mathcal{G}_{\mu\nu}^\alpha \tilde{\mathcal{G}}^{\alpha\mu\nu}, \end{aligned} \quad (8)$$

where $\tilde{\mathcal{G}}^{\mu\nu} \equiv \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} \mathcal{G}_{\rho\sigma}$. It contains the kinetic terms for all the fields, the theta term of QCD, the mass terms

¹ The discussion on neutrino masses will not be treated here. See Ref. [18, 52] for details.

for the EW gauge bosons, the Yukawa interactions and the Higgs scalar potencial, whose specific form depends on the model under consideration [53]. The functions $\mathcal{Y}_{Q,L}(h)$ appearing in the Yukawa couplings are written in a compact notation:

$$\begin{aligned}\mathcal{Y}_Q(h) &\equiv \text{diag} \left(\sum_n Y_U^{(n)} \frac{h^n}{v^n}, \sum_n Y_D^{(n)} \frac{h^n}{v^n} \right), \\ \mathcal{Y}_L(h) &\equiv \text{diag} \left(0, \sum_n Y_\ell^{(n)} \frac{h^n}{v^n} \right).\end{aligned}\quad (9)$$

The $n = 0$ terms yield fermion masses, while the higher orders describe the interactions with n insertions of the Higgs field h , accounting in general for non-aligned contributions. The structure of these terms, however, is customary simplified in phenomenological analyses, assuming Yukawa interactions aligned with fermion masses, i.e. $Y_{U,D,\ell}^{(n)} = Y_{U,D,\ell}^{(0)}$ [11, 14, 16, 17]. The same assumption will be adopted here and in consequence

$$\begin{aligned}\mathcal{Y}_Q(h) &= \text{diag} \left(Y_U^{(0)} \mathcal{F}_U(h), Y_D^{(0)} \mathcal{F}_D(h) \right), \\ \mathcal{Y}_L(h) &= \text{diag} \left(0, Y_\ell^{(0)} \mathcal{F}_\ell(h) \right).\end{aligned}\quad (10)$$

The function $\mathcal{F}_C(h)$ appearing in the GB kinetic term is typically expanded as

$$\mathcal{F}_C(h) = 1 + 2a_C \frac{h}{v} + b_C \frac{h^2}{v^2} + \dots \quad (11)$$

It is convenient to make explicit the beyond SM part of the coefficients a_C , b_C , using the notation

$$a_C = 1 + \Delta a_C, \quad b_C = 1 + \Delta b_C, \quad (12)$$

where Δa_C , Δb_C are assumed to be of the same order as the coefficients accompanying the operators appearing in the second part of the Lagrangian $\Delta\mathcal{L}_h$, which accounts for new interactions and for deviations from the leading order (LO) one.

$\mathcal{F}(h)$ functions could be inserted into the kinetic terms for fermions and the physical Higgs, but their contributions can be reabsorbed inside the generic functions $\mathcal{F}_C(h)$ and $\mathcal{Y}_{Q,L}(h)$, as discussed in Ref. [17]. The kinetic terms of the gauge bosons are also free from any $\mathcal{F}(h)$ in this LO Lagrangian, assuming that the transverse components of the gauge bosons do not couple strongly to the EWSB sector.

Following Refs. [14, 17], the second part of the Lagrangian in Eq. (7) contains all the operators necessary for reabsorbing 1-loop divergences arising from the renormalisation of $\mathcal{L}_h^{(0)}$ and operators with similar suppression. Adopting the notation used in Refs. [38, 39] and the NDA normalisation [20–22], $\Delta\mathcal{L}_h$ can be written as the sum of several terms: focussing only on the CP-even

bosonic ones, one can write

$$\begin{aligned}\Delta\mathcal{L}_h &= c_T \mathcal{P}_T \mathcal{F}_T(h) + c_B \mathcal{P}_B \mathcal{F}_B(h) + \\ &+ c_W \mathcal{P}_W \mathcal{F}_W(h) + c_G \mathcal{P}_G \mathcal{F}_G(h) + \\ &+ c_H \mathcal{P}_H \mathcal{F}_H(h) + c_{\square H} \mathcal{P}_{\square H} \mathcal{F}_{\square H}(h) + \\ &+ c_{\Delta H} \mathcal{P}_{\Delta H} \mathcal{F}_{\Delta H}(h) + c_{DH} \mathcal{P}_{DH} \mathcal{F}_{DH}(h) + \\ &+ c_{WWW} \mathcal{P}_{WWW} \mathcal{F}_{WWW}(h) + \\ &+ c_{GGG} \mathcal{P}_{GGG} \mathcal{F}_{GGG}(h) + c_{DB} \mathcal{P}_{DB} \mathcal{F}_{DB}(h) + \\ &+ c_{DW} \mathcal{P}_{DW} \mathcal{F}_{DW}(h) + c_{DG} \mathcal{P}_{DG} \mathcal{F}_{DG}(h) + \\ &+ \sum_{i=1}^{26} c_i \mathcal{P}_i \mathcal{F}_i(h),\end{aligned}\quad (13)$$

where the parameters c_i are free coefficients smaller than 1, according to the Naive Dimensional Analysis formulation [20–22]. The term \mathcal{P}_T is a custodial-breaking two-derivative operator,

$$\mathcal{P}_T = \frac{v^2}{4} \text{Tr}(\mathbf{T}\mathbf{V}_\mu) \text{Tr}(\mathbf{T}\mathbf{V}^\mu), \quad (14)$$

that traditionally is inserted at the LO, but whose coefficient is such strongly constrained ($\lesssim 10^{-2}$) from the EW precision parameter T that is customary listed in $\Delta\mathcal{L}_h$. The three operators \mathcal{P}_B , \mathcal{P}_W and \mathcal{P}_G ,

$$\begin{aligned}\mathcal{P}_B &= -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} \\ \mathcal{P}_W &= -\frac{1}{4} W_{\mu\nu}^a W^{a\mu\nu} \\ \mathcal{P}_G &= -\frac{1}{4} \mathcal{G}_{\mu\nu}^\alpha \mathcal{G}^{\alpha\mu\nu},\end{aligned}\quad (15)$$

contain the field strengths for the SM gauge bosons and, once multiplied by the corresponding $\mathcal{F}(h)$, describe the interactions between h and the transverse components of the gauge bosons.

The four operators \mathcal{P}_H , $\mathcal{P}_{\square H}$, $\mathcal{P}_{\Delta H}$ and \mathcal{P}_{DH} ,

$$\begin{aligned}\mathcal{P}_H &= \frac{1}{2} (\partial_\mu h) (\partial^\mu h) \\ \mathcal{P}_{\square H} &= \frac{1}{\Lambda^2} (\square h)^2 \\ \mathcal{P}_{\Delta H} &= \frac{4\pi}{\Lambda^3} (\partial_\mu h) (\partial^\mu h) (\square h) \\ \mathcal{P}_{DH} &= \frac{(4\pi)^2}{\Lambda^4} ((\partial_\mu h) (\partial^\mu h))^2,\end{aligned}\quad (16)$$

describe pure Higgs couplings with two and four derivatives.

The five pure-gauge operators \mathcal{P}_{WWW} , \mathcal{P}_{GGG} , \mathcal{P}_{DB} ,

\mathcal{P}_{DW} and \mathcal{P}_{DG} ,

$$\begin{aligned}
\mathcal{P}_{WWW} &= \frac{4\pi\varepsilon_{abc}}{\Lambda^2} W_\mu^{a\nu} W_\nu^{b\rho} W_\rho^{c\mu}, \\
\mathcal{P}_{GGG} &= \frac{4\pi f_{\alpha\beta\gamma}}{\Lambda^2} G_\mu^{\alpha\nu} G_\nu^{\beta\rho} G_\rho^{\gamma\mu} \\
\mathcal{P}_{DB} &= \frac{1}{\Lambda^2} (\partial^\mu B_{\mu\nu}) (\partial_\rho B^{\rho\nu}) \\
\mathcal{P}_{DW} &= \frac{1}{\Lambda^2} (\mathcal{D}^\mu W_{\mu\nu})^a (\mathcal{D}_\rho W^{\rho\nu})^a \\
\mathcal{P}_{DG} &= \frac{1}{\Lambda^2} (\mathcal{D}^\mu G_{\mu\nu})^\alpha (\mathcal{D}_\rho G^{\rho\nu})^\alpha,
\end{aligned} \tag{17}$$

with ε_{abc} and $f_{\alpha\beta\gamma}$ the structure constants of $SU(2)_L$ and

$SU(3)_c$ respectively, are typically listed at higher orders in the chiral perturbation theory, but in the HEFT they should be inserted in $\Delta\mathcal{L}_h$: indeed, they have the same suppressions of $\mathcal{P}_{\square H}$ in Eq. (16) (and of four-fermion operators, if fermion couplings would be considered – see Ref. [17]). This is consistent with the fact that the HEFT merges together the traditional expansion in canonical dimensions and the expansion in derivatives of the chiral perturbation theory (see Ref. [22] for details).

The rest of the terms are four-derivative operators defined as

$$\begin{aligned}
\mathcal{P}_1 &= B_{\mu\nu} \text{Tr}(\mathbf{T} \mathbf{W}^{\mu\nu}) \\
\mathcal{P}_2 &= \frac{i}{4\pi} B_{\mu\nu} \text{Tr}(\mathbf{T} [\mathbf{V}^\mu, \mathbf{V}^\nu]) \\
\mathcal{P}_3 &= \frac{i}{4\pi} \text{Tr}(\mathbf{W}_{\mu\nu} [\mathbf{V}^\mu, \mathbf{V}^\nu]) \\
\mathcal{P}_4 &= \frac{i}{\Lambda} B_{\mu\nu} \text{Tr}(\mathbf{T} \mathbf{V}^\mu) \partial^\nu h \\
\mathcal{P}_5 &= \frac{i}{\Lambda} \text{Tr}(\mathbf{W}_{\mu\nu} \mathbf{V}^\mu) \partial^\nu h \\
\mathcal{P}_6 &= \frac{1}{(4\pi)^2} (\text{Tr}(\mathbf{V}_\mu \mathbf{V}^\mu))^2 \\
\mathcal{P}_7 &= \frac{1}{4\pi\Lambda} \text{Tr}(\mathbf{V}_\mu \mathbf{V}^\mu) \partial_\nu \partial^\nu h \\
\mathcal{P}_8 &= \frac{1}{\Lambda^2} \text{Tr}(\mathbf{V}_\mu \mathbf{V}_\nu) \partial^\mu h \partial^\nu h \\
\mathcal{P}_9 &= \frac{1}{(4\pi)^2} \text{Tr}((\mathcal{D}_\mu \mathbf{V}^\mu)^2) \\
\mathcal{P}_{10} &= \frac{1}{4\pi\Lambda} \text{Tr}(\mathbf{V}_\nu \mathcal{D}_\mu \mathbf{V}^\mu) \partial^\nu h \\
\mathcal{P}_{11} &= \frac{1}{(4\pi)^2} (\text{Tr}(\mathbf{V}_\mu \mathbf{V}_\nu))^2 \\
\mathcal{P}_{12} &= (\text{Tr}(\mathbf{T} \mathbf{W}_{\mu\nu}))^2 \\
\mathcal{P}_{13} &= \frac{i}{4\pi} \text{Tr}(\mathbf{T} \mathbf{W}_{\mu\nu}) \text{Tr}(\mathbf{T} [\mathbf{V}^\mu, \mathbf{V}^\nu]) \\
\mathcal{P}_{14} &= \frac{\varepsilon_{\mu\nu\rho\lambda}}{4\pi} \text{Tr}(\mathbf{T} \mathbf{V}^\mu) \text{Tr}(\mathbf{V}^\nu \mathbf{W}^{\rho\lambda}) \\
\mathcal{P}_{15} &= \frac{1}{(4\pi)^2} \text{Tr}(\mathbf{T} \mathcal{D}_\mu \mathbf{V}^\mu) \text{Tr}(\mathbf{T} \mathcal{D}_\nu \mathbf{V}^\nu) \\
\mathcal{P}_{16} &= \frac{1}{(4\pi)^2} \text{Tr}([\mathbf{T}, \mathbf{V}_\nu] \mathcal{D}_\mu \mathbf{V}^\mu) \text{Tr}(\mathbf{T} \mathbf{V}^\nu) \\
\mathcal{P}_{17} &= \frac{i}{\Lambda} \text{Tr}(\mathbf{T} \mathbf{W}_{\mu\nu}) \text{Tr}(\mathbf{T} \mathbf{V}^\mu) \partial^\nu h \\
\mathcal{P}_{18} &= \frac{1}{4\pi\Lambda} \text{Tr}(\mathbf{T} [\mathbf{V}_\mu, \mathbf{V}_\nu]) \text{Tr}(\mathbf{T} \mathbf{V}^\mu) \partial^\nu h \\
\mathcal{P}_{19} &= \frac{1}{4\pi\Lambda} \text{Tr}(\mathbf{T} \mathcal{D}_\mu \mathbf{V}^\mu) \text{Tr}(\mathbf{T} \mathbf{V}_\nu) \partial^\nu h \\
\mathcal{P}_{20} &= \frac{1}{\Lambda^2} \text{Tr}(\mathbf{V}_\mu \mathbf{V}^\mu) \partial_\nu h \partial^\nu h \\
\mathcal{P}_{21} &= \frac{1}{\Lambda^2} (\text{Tr}(\mathbf{T} \mathbf{V}_\mu))^2 \partial_\nu h \partial^\nu h \\
\mathcal{P}_{22} &= \frac{1}{\Lambda^2} \text{Tr}(\mathbf{T} \mathbf{V}_\mu) \text{Tr}(\mathbf{T} \mathbf{V}_\nu) \partial^\mu h \partial^\nu h \\
\mathcal{P}_{23} &= \frac{1}{(4\pi)^2} \text{Tr}(\mathbf{V}_\mu \mathbf{V}^\mu) (\text{Tr}(\mathbf{T} \mathbf{V}_\nu))^2 \\
\mathcal{P}_{24} &= \frac{1}{(4\pi)^2} \text{Tr}(\mathbf{V}_\mu \mathbf{V}_\nu) \text{Tr}(\mathbf{T} \mathbf{V}^\mu) \text{Tr}(\mathbf{T} \mathbf{V}^\nu) \\
\mathcal{P}_{25} &= \frac{1}{4\pi\Lambda} (\text{Tr}(\mathbf{T} \mathbf{V}_\mu))^2 \partial_\nu h \partial^\nu h \\
\mathcal{P}_{26} &= \frac{1}{(4\pi)^2} (\text{Tr}(\mathbf{T} \mathbf{V}_\mu) \text{Tr}(\mathbf{T} \mathbf{V}_\nu))^2,
\end{aligned} \tag{18}$$

where the normalisation follows the NDA of Refs. [22].

The operators of this list containing the scalar chiral field \mathbf{T} , not in association with the gauge field strength $B_{\mu\nu}$, represent sources of custodial symmetry breaking beyond those of the SM: they are $\mathcal{P}_{12} - \mathcal{P}_{26}$, except for \mathcal{P}_{20} .

Once specifying an underlying scenario, it is then possible to write the Wilson coefficients c_i and the functions $\mathcal{F}_i(h)$ of the HEFT in terms of the parameters of the high-energy Lagrangian. In Refs. [38, 39], a generic

Composite Higgs model has been considered and the corresponding Lagrangian at high-energy has been projected at low energy on the HEFT. The results can be read in Tables 1 and 2 of Ref. [38] and Table 1 of Ref. [39]. This exercise has pointed out that, in traditional Composite Higgs models, gauge-Higgs couplings are correlated to pure gauge ones, similarly to what happens in the SMEFT, although with a few differences: this follows from the fact that the Higgs field belongs to a representation that contains the $SU(2)_L$ -doublet one [54]. The SMEFT is then a good low-energy description of Com-

posite Higgs models at the very first order, but deviations arise at higher orders. As a result, one could disentangle an elementary from a composite Higgs comparing the same gauge interactions but with different Higgs legs.

The dilaton, that will be the subject of the next section, is a singlet under $SU(2)_L$ and therefore distinct phenomenological features are expected: indeed, the EW doublet or singlet representation of the Higgs field is encoded into correlations/decorrelations between pure-gauge and gauge-Higgs couplings.

III. THE HIGGS-LIKE DILATON

The dilaton arises as a GB of the spontaneous breaking of the scale invariance. This mechanism is typical of models where the EWSB and the scale symmetry breaking do not coincide, such as when the EWSB is strongly coupled: the scale symmetry breaking scale f is typically larger than the EW VEV v and the states arising from the scale symmetry breaking do not need to coincide with those responsible for the EWSB mechanism. Explicit realisations are theories of walking technicolor [55–57], and of Randall-Sundrum extra-dimension constructions [58–61].

Considering a generic Lagrangian written as the sum of distinct operators $\mathcal{O}_i(x)$ with canonical dimension $[\mathcal{O}_i] \equiv \mathbf{d}_i$ and with coefficient $g_i(\mu)$, where μ is the reference scale,

$$\mathcal{L} = \sum_i g_i(\mu) \mathcal{O}_i(x), \quad (19)$$

scale transformations $x^\mu \rightarrow e^\lambda x^\mu$ give

$$\begin{aligned} \mathcal{O}_i(x) &\rightarrow e^{\lambda \mathbf{d}_i} \mathcal{O}_i(e^\lambda x), \\ \mu &\rightarrow e^{-\lambda} \mu, \end{aligned} \quad (20)$$

that leads to the variation of the Lagrangian

$$\delta \mathcal{L} = \sum_i g_i(\mu) (\mathbf{d}_i + x^\mu \partial_\mu) \mathcal{O}_i(x) + \sum_i \beta_i(g) \mathcal{O}_i(x), \quad (21)$$

where

$$\beta_i(g) = \mu \frac{\partial g_i(\mu)}{\partial \mu}, \quad (22)$$

are the beta functions of the couplings g_i . The Lagrangian is scale invariant when $\delta \mathcal{L} = 0$, that corresponds to $\beta_i(g) = 0$, i.e. the couplings do not depend on the scale considered, and to the canonical dimensions of the operators satisfying to $\mathbf{d}_i = 4$ (by using integration by parts, $(\mathbf{d}_i + x^\mu \partial_\mu) \mathcal{O}_i(x) \rightarrow (\mathbf{d}_i - 4) \mathcal{O}_i(x)$).

In the SM, fermion ($\mathbf{d} = 3$) and scalar ($\mathbf{d} = 2$) mass terms explicitly violate the scale symmetry. However, as adopted in the context of the so-called Minimal Flavour Violation [62–75], or more in general in flavour models [76–91], a strategy to reestablish scale invariance consists in enlarging the spectrum by the addition of a scalar

field, $\chi(x)$, that transforms under the scale symmetry: insertion of powers of this scalar field in the Lagrangian operators then allows to recover exactly scale invariance. Once it develops a vacuum expectation value (VEV), fermion and scalar masses are correctly described and the scale symmetry is spontaneously broken with the arising of the corresponding GB, the dilaton.

Defining the scale transformation law for the auxiliary scalar field $\chi(x)$ – sometimes also called conformal compensator – as

$$\chi(x) \rightarrow e^\lambda \chi(e^\lambda x), \quad (23)$$

scale symmetry invariance is recovered performing the following replacement for the couplings g_i in Eq. (19):

$$g_i(\mu) \rightarrow g_i \left(\mu \frac{\chi}{f} \right) \left(\frac{\chi}{f} \right)^{4-\mathbf{d}_i}. \quad (24)$$

The scale f in the previous equation is identified with the VEV of $\chi(x)$, $f = \langle \chi \rangle$, and represents the scale of the symmetry breaking. Following the traditional notation for Goldstone bosons, the conformal compensator $\chi(x)$ can be parametrised as

$$\chi(x) = f e^{\sigma(x)/f}, \quad (25)$$

making explicit the GB σ , whose scale transformation is non-linear, $\sigma(x)/f \rightarrow \sigma(e^\lambda x)/f + \lambda$.

To simplify the notation, the explicit dependence of $\chi(x)$ and $\sigma(x)$ on x will be suppressed in what follows.

A. The DEFT Lagrangian

The Dilaton Effective Field Theory (DEFT) Lagrangian is constructed in a very similar way to the HEFT one, by exchanging the Higgs field h with the conformal compensator χ , or the dilaton field σ , and by requiring scale symmetry invariance at the classical level by implementing Eq. (24). Following Sect. II, the DEFT Lagrangian can be written as the sum of two terms,

$$\mathcal{L}_{\text{DEFT}} \equiv \mathcal{L}_\chi^{(0)} + \Delta \mathcal{L}_\chi, \quad (26)$$

where the first term is very similar to $\mathcal{L}_h^{(0)}$ of Eq. (8):

$$\begin{aligned} \mathcal{L}_\chi^{(0)} = & -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W_{\mu\nu}^a W^{a\mu\nu} - \frac{1}{4} \mathcal{G}_{\mu\nu}^\alpha \mathcal{G}^{\alpha\mu\nu} + \\ & + \frac{1}{2} \partial_\mu \chi \partial^\mu \chi - \frac{v^2}{4} \left(\frac{\chi}{f} \right)^2 \text{Tr}(\mathbf{V}_\mu \mathbf{V}^\mu) - V(\chi) + \\ & + i \bar{Q}_L \not{D} Q_L + i \bar{Q}_R \not{D} Q_R + i \bar{L}_L \not{D} L_L + i \bar{L}_R \not{D} L_R + \\ & - \frac{v}{\sqrt{2} f} (\bar{Q}_L \mathbf{U} \mathcal{Y}_Q Q_R + \text{h.c.}) + \\ & - \frac{v}{\sqrt{2} f} (\bar{L}_L \mathbf{U} \mathcal{Y}_L L_R + \text{h.c.}) + \\ & - \frac{g_s^2}{(4\pi)^2} \lambda_s \mathcal{G}_{\mu\nu}^\alpha \tilde{\mathcal{G}}^{\alpha\mu\nu}. \end{aligned} \quad (27)$$

where

$$\mathcal{Y}_Q \equiv \text{diag} \left(Y_U^{(0)}, Y_D^{(0)} \right), \quad \mathcal{Y}_L \equiv \text{diag} \left(0, Y_\ell^{(0)} \right). \quad (28)$$

This Lagrangian contains the kinetic and mass terms for all the fields of the spectrum, except the mass term for the dilaton, and leading interactions among SM gauge bosons and fermions and the dilaton. As for the HEFT, no interaction is present at this order between the transverse components of the EW gauge bosons and the dilaton: here, however, in contrast to the HEFT, it does not follow any assumption, but it is simply due to the fact that the kinetic terms are already scale symmetry invariant and in consequence do not allow any additional insertion of (χ/f) . The same holds for the kinetic terms for fermions and for the dilaton. On the contrary, EW gauge boson mass and Yukawa terms require the presence of χ in order to guarantee the scale symmetry invariance. Finally, $V(\chi)$ contains non-derivative self-couplings of the conformal compensator χ : $V(\chi)$ may contain scale symmetry breaking factors, whose magnitude depends on the underlying theory considered. To avoid to enter into details of specific realisation, the explicit form of $V(\chi)$ will not be discussed here and it will only be assumed that $V(\chi)$ is minimised by $\langle \chi \rangle = f$.

Rewriting $\mathcal{L}_\chi^{(0)}$ in terms of the dilaton field σ through Eq. (25), it follows that the dilaton kinetic term is not canonically normalised:

$$\begin{aligned} \frac{1}{2} \partial_\mu \chi \partial^\mu \chi &= \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma e^{2\sigma/f} \\ &= \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma \left(1 + \frac{2\sigma}{f} + \frac{2\sigma^2}{f^2} + \dots \right), \end{aligned} \quad (29)$$

where in the last equality the Taylor series expansion of the exponential has been explicitly written and dots stand for higher powers in σ/f . Performing the following redefinition on the dilaton field,

$$\sigma = f \ln \left(\frac{\bar{\chi}}{f} + 1 \right), \quad (30)$$

canonical normalised kinetic terms are recovered, where $\bar{\chi}$ are the fluctuations of χ around its VEV. The resulting leading Lagrangian after this redefinition looks exactly as in Eq. (27), except for a few terms, where χ insertions are substituted by insertions of $\bar{\chi} + f$:

$$\partial_\mu \chi \partial^\mu \chi \rightarrow \partial_\mu \bar{\chi} \partial^\mu \bar{\chi}, \quad (31a)$$

$$\left(\frac{\chi}{f} \right)^2 \text{Tr}(\mathbf{V}_\mu \mathbf{V}^\mu) \rightarrow \left(1 + \frac{\bar{\chi}}{f} \right)^2 \text{Tr}(\mathbf{V}_\mu \mathbf{V}^\mu), \quad (31b)$$

$$\frac{\chi}{f} \bar{Q}_L \mathbf{U} \mathcal{Y}_Q Q_R \rightarrow \left(1 + \frac{\bar{\chi}}{f} \right) \bar{Q}_L \mathbf{U} \mathcal{Y}_Q Q_R, \quad (31c)$$

$$\frac{\chi}{f} \bar{L}_L \mathbf{U} \mathcal{Y}_L L_R \rightarrow \left(1 + \frac{\bar{\chi}}{f} \right) \bar{L}_L \mathbf{U} \mathcal{Y}_L L_R. \quad (31d)$$

It is interesting to note that Eq. (31b), once assuming $f = v$, coincides with the SM result, that is Eq. (11)

for $a_C = 1 = b_C$ and the rest of coefficients vanishing. This well-known result (see for example Ref. [32]) points out that only a partial unitarisation of the SM amplitudes occurs in the DEFT as long as $f > v$. The full unitarisation should then be accomplished by new degrees of freedom, that are expected to arise at the scale Λ , for example assuming an underlying strong dynamics.

The second term of the DEFT Lagrangian in Eq. (26) contains couplings of the dilaton that go beyond the SM-like ones of Eqs. (27) and (31), and therefore could provide discriminating signals for shedding light on the Higgs nature. Restricting only on the CP-even bosonic couplings, $\Delta \mathcal{L}_\chi$ can be written as the sum of distinct operators:

$$\begin{aligned} \Delta \mathcal{L}_\chi &= d_T \mathcal{P}_T \mathcal{F}_T(\chi) + d_B \mathcal{P}_B \mathcal{F}_B(\chi) + \\ &+ d_W \mathcal{P}_W \mathcal{F}_W(\chi) + d_G \mathcal{P}_G \mathcal{F}_G(\chi) + \\ &+ d_{\square H} \mathcal{P}_{\square H} \mathcal{F}_{\square H}(\chi) + d_{\Delta H} \mathcal{P}_{\Delta H} \mathcal{F}_{\Delta H}(\chi) \\ &+ d_{DH} \mathcal{P}_{DH} \mathcal{F}_{DH}(\chi) + \\ &+ d_{WWW} \mathcal{P}_{WWW} \mathcal{F}_{WWW}(\chi) + \\ &+ d_{GGG} \mathcal{P}_{GGG} \mathcal{F}_{GGG}(\chi) + d_{DB} \mathcal{P}_{DB} \mathcal{F}_{DB}(h) + \\ &+ d_{DW} \mathcal{P}_{DW} \mathcal{F}_{DW}(h) + d_{DG} \mathcal{P}_{DG} \mathcal{F}_{DG}(h) + \\ &+ \sum_{i=1}^{26} d_i \mathcal{P}_i \mathcal{F}_i(\chi), \end{aligned} \quad (32)$$

where the parameters d_i are free coefficients smaller than 1 – in particular products of two or more operator coefficients d_i will be neglected in the phenomenological analysis of the next sections. Moreover, the operators \mathcal{P}_i are similar to the list of operators \mathcal{P}_i in Eqs. (13)–(18) for the HEFT, but with the proper modifications for the dilaton case discussed in this section. On the other side, the functions $\mathcal{F}_i(\chi)$, differently from the functions $\mathcal{F}(h)$ of the HEFT, are *not* arbitrary functions of the dilaton field, but depends on the specific operator considered.

In the first line of Eq. (32),

$$\begin{aligned} \mathcal{P}_T \mathcal{F}_T(\chi) &= \frac{v^2}{4} \text{Tr}(\mathbf{T} \mathbf{V}_\mu) \text{Tr}(\mathbf{T} \mathbf{V}^\mu) \frac{\chi^2}{f^2} \\ &\rightarrow \frac{v^2}{4} \text{Tr}(\mathbf{T} \mathbf{V}_\mu) \text{Tr}(\mathbf{T} \mathbf{V}^\mu) \left(1 + \frac{\bar{\chi}}{f} \right)^2, \end{aligned} \quad (33)$$

where the operator in the first (second) line is written before (after) moving to the basis of canonical kinetic term for the dilaton, Eq. (30). Moreover, the terms in the brackets correspond to the function $\mathcal{F}_T(\chi)$. The $\bar{\chi}$ -independent term provides a tree-level contribution to the EW precision parameter T , exactly as for the operator \mathcal{P}_T in Eq. (14), and therefore the coefficient d_T is constrained to be at most at the per cent level, such as c_T .

The four operators \mathcal{P}_B , \mathcal{P}_W , \mathcal{P}_G and \mathcal{P}_1 describe the interaction of the dilaton with the transverse components

of the gauge bosons:

$$\begin{aligned}
\mathcal{P}_B \mathcal{F}_B(\chi) &= -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} \frac{\bar{\chi}}{f} \\
\mathcal{P}_W \mathcal{F}_W(\chi) &= -\frac{1}{4} W_{\mu\nu}^a W^{a\mu\nu} \frac{\bar{\chi}}{f} \\
\mathcal{P}_G \mathcal{F}_G(\chi) &= -\frac{1}{4} G_{\mu\nu}^\alpha G^{\alpha\mu\nu} \frac{\bar{\chi}}{f} \\
\mathcal{P}_1 \mathcal{F}_1(\chi) &= B_{\mu\nu} \text{Tr}(\mathbf{T} \mathbf{W}^{\mu\nu}) \frac{\bar{\chi}}{f},
\end{aligned} \tag{34}$$

where the last factor $\bar{\chi}/f$ in each operator represents the functions $\mathcal{F}_i(\chi)$. From the point of view of the scale invariance, insertion of $\frac{\bar{\chi}}{f}$ would not be necessary. However, with mild assumptions on the underlying theory, an in particular just assuming that the SM gauge symmetry is part of the conformal sector, 1-loop contributions to these couplings naturally arise: the explicit computations have been computed in Ref. [32]²; generic couplings are considered in Eq. (34) in order to keep general and model independent the effective Lagrangian.

The three operators $\mathcal{P}_{\square\chi}$, $\mathcal{P}_{\Delta\chi}$ and $\mathcal{P}_{D\chi}$ have a similar structure to the one of the HEFT operators $\mathcal{P}_{\square H}$, $\mathcal{P}_{\Delta H}$ and \mathcal{P}_{DH} defined in Eq. (16):

$$\begin{aligned}
\mathcal{P}_{\square\chi} \mathcal{F}_{\square\chi}(\chi) &= \frac{1}{(4\pi)^2 \chi^2} (\square\chi)^2 \\
&\rightarrow \frac{1}{\Lambda^2} (\square\bar{\chi})^2 \left(1 - \frac{2\bar{\chi}}{f} + \frac{3\bar{\chi}^2}{f^2} + \dots\right) \\
\mathcal{P}_{\Delta\chi} \mathcal{F}_{\Delta\chi}(\chi) &= \frac{1}{(4\pi)^2 \chi^3} (\partial_\mu \chi \partial^\mu \chi) \square\chi \\
&\rightarrow \frac{4\pi}{\Lambda^3} (\partial_\mu \bar{\chi} \partial^\mu \bar{\chi}) (\square\bar{\chi}) \left(1 - \frac{3\bar{\chi}}{f} + \frac{6\bar{\chi}^2}{f^2} + \dots\right) \\
\mathcal{P}_{D\chi} \mathcal{F}_{D\chi}(\chi) &= \frac{1}{(4\pi)^2 \chi^4} (\partial_\mu \chi \partial^\mu \chi)^2 \\
&\rightarrow \frac{(4\pi)^2}{\Lambda^4} (\partial_\mu \bar{\chi} \partial^\mu \bar{\chi})^2 \left(1 - \frac{4\bar{\chi}}{f} + \frac{10\bar{\chi}^2}{f^2} + \dots\right),
\end{aligned} \tag{35}$$

where $\Lambda = 4\pi f$ has been used and the last factors inside the brackets represent the functions $\mathcal{F}_i(\chi)$.

A first difference between Eqs. (16) and (35) is the absence of an equivalent operator to \mathcal{P}_H : being $\partial_\mu \chi \partial^\mu \chi$ already of $\mathbf{d} = 4$, no additional χ^n insertion is necessary for scale invariance; moreover, differently from what discussed for the operators in Eq. (34), no physical³ 1-loop correction affects this operator.

A second important difference appears in the functions $\mathcal{F}_i(h)$ and $\mathcal{F}_i(\chi)$: the first ones are completely generic polynomials of h/v in the HEFT; for the dilaton case, only specific numerical coefficients multiply the distinct factors $\bar{\chi}/f$. When the experimental sensitivity on the involved observables will be high enough, the identification of these numerical coefficients could tell a Higgs-like dilaton from other possibilities, as indeed they are different in the SMEFT case, where $\mathcal{F}_i(h)$ can only be powers of $(1 + h/v)^2$, or in CH models where $\mathcal{F}_i(h)$ are trigonometric functions of h/f : this will be further discussed in the next sections.

The normalisation of the operators in Eq. (35) has been fixed such to match the one of the operators in Eq. (16): although this is not necessary from the EFT point of view, with this choice one recovers the Naive Dimensional Analysis normalisation, which tells that the theory enters into a strong interacting regime⁴ when $d_i = 1$.

The five operators \mathcal{P}_{WWW} , \mathcal{P}_{GGG} , \mathcal{P}_{DB} , \mathcal{P}_{DW} and \mathcal{P}_{DG} in the last lines of Eq. (32) are defined as

$$\begin{aligned}
\mathcal{P}_{WWW} \mathcal{F}_{WWW}(\chi) &= \frac{\varepsilon_{abc}}{4\pi \chi^2} W_\mu^{a\nu} W_\nu^{b\rho} W_\rho^{c\mu} \\
&\rightarrow \frac{4\pi \varepsilon_{abc}}{\Lambda^2} W_\mu^{a\nu} W_\nu^{b\rho} W_\rho^{c\mu} \left(1 - \frac{2\bar{\chi}}{f} + \frac{3\bar{\chi}^2}{f^2} + \dots\right) \\
\mathcal{P}_{GGG} \mathcal{F}_{GGG}(\chi) &= \frac{f_{\alpha\beta\gamma}}{4\pi \chi^2} G_\mu^{\alpha\nu} G_\nu^{\beta\rho} G_\rho^{\gamma\mu} \\
&\rightarrow \frac{4\pi f_{\alpha\beta\gamma}}{\Lambda^2} G_\mu^{\alpha\nu} G_\nu^{\beta\rho} G_\rho^{\gamma\mu} \left(1 - \frac{2\bar{\chi}}{f} + \frac{3\bar{\chi}^2}{f^2} + \dots\right) \\
\mathcal{P}_{DB} \mathcal{F}_{DB}(\chi) &= \frac{1}{(4\pi)^2 \chi^2} (\partial^\mu B_{\mu\nu}) (\partial_\rho B^{\rho\nu}) \\
&\rightarrow \frac{1}{\Lambda^2} (\partial^\mu B_{\mu\nu})^\alpha (\partial_\rho B^{\rho\nu})^\alpha \left(1 - \frac{2\bar{\chi}}{f} + \frac{3\bar{\chi}^2}{f^2} + \dots\right) \\
\mathcal{P}_{DW} \mathcal{F}_{DW}(\chi) &= \frac{1}{(4\pi)^2 \chi^2} (\mathcal{D}^\mu W_{\mu\nu})^a (\mathcal{D}_\rho W^{\rho\nu})^a \\
&\rightarrow \frac{1}{\Lambda^2} (\mathcal{D}^\mu W_{\mu\nu})^a (\mathcal{D}_\rho W^{\rho\nu})^a \left(1 - \frac{2\bar{\chi}}{f} + \frac{3\bar{\chi}^2}{f^2} + \dots\right) \\
\mathcal{P}_{DG} \mathcal{F}_{DG}(\chi) &= \frac{1}{(4\pi)^2 \chi^2} (\mathcal{D}^\mu G_{\mu\nu})^\alpha (\mathcal{D}_\rho G^{\rho\nu})^\alpha \\
&\rightarrow \frac{1}{\Lambda^2} (\mathcal{D}^\mu G_{\mu\nu})^\alpha (\mathcal{D}_\rho G^{\rho\nu})^\alpha \left(1 - \frac{2\bar{\chi}}{f} + \frac{3\bar{\chi}^2}{f^2} + \dots\right),
\end{aligned} \tag{36}$$

² Ref. [32] only considered contributions to the massless gauge bosons. The same reasoning applies to couplings of the dilaton to massive gauge bosons, $WW\bar{\chi}$ and $ZZ\bar{\chi}$, and to the $Z\gamma\bar{\chi}$ coupling.

³ Any non-derivative polynomial of χ could be redefined away by a redefinition of the field χ . This would translate in a modification, proportional to the corresponding operator coefficient, of all the other χ couplings in the Lagrangian. As the product of two or more operator coefficients are neglected, this redefinition would not have any impact. See App. B in Ref. [17] for further details.

⁴ A theory enters into a strong interacting regime when loop corrections to the Wilson coefficient of a generic operator turn out to be (at least) equally important than the initial value of the Wilson coefficient itself. The Naive Dimensional Analysis normalisation used in Eq. (13) for HEFT identifies this phase transition with the condition of having operator coefficient equal or larger than 1. A shortcut to ensure this condition for the dilaton case is to profit of the similarities between the DEFT and the HEFT Lagrangians, when identifying the scale Λ as the same scale in both the frameworks. The numerical factors in the operators in Eq. (35) – and in the equations which follow – has been fixed in this way.

where the last factors inside the brackets represent the functions $\mathcal{F}_i(\chi)$.

The remaining terms entering the Lagrangian $\Delta\mathcal{L}_\chi$ are four-derivative operators and are defined as follows:

$$\begin{aligned}
\mathcal{P}_2\mathcal{F}_2(\chi) &= \frac{i}{4\pi} B_{\mu\nu} \text{Tr}(\mathbf{T}[\mathbf{V}^\mu, \mathbf{V}^\nu]) \\
\mathcal{P}_3\mathcal{F}_3(\chi) &= \frac{i}{4\pi} \text{Tr}(\mathbf{W}_{\mu\nu}[\mathbf{V}^\mu, \mathbf{V}^\nu]) \\
\mathcal{P}_4\mathcal{F}_4(\chi) &= \frac{i}{4\pi\chi} B_{\mu\nu} \text{Tr}(\mathbf{T}\mathbf{V}^\mu) \partial^\nu \chi \\
&\rightarrow \frac{i}{\Lambda} B_{\mu\nu} \text{Tr}(\mathbf{T}\mathbf{V}^\mu) \partial^\nu \bar{\chi} \left(1 - \frac{\bar{\chi}}{f} + \frac{\bar{\chi}^2}{f^2} + \dots\right) \\
\mathcal{P}_5\mathcal{F}_5(\chi) &= \frac{i}{4\pi\chi} \text{Tr}(\mathbf{W}_{\mu\nu}\mathbf{V}^\mu) \partial^\nu \chi \\
&\rightarrow \frac{i}{\Lambda} \text{Tr}(\mathbf{W}_{\mu\nu}\mathbf{V}^\mu) \partial^\nu \bar{\chi} \left(1 - \frac{\bar{\chi}}{f} + \frac{\bar{\chi}^2}{f^2} + \dots\right) \\
\mathcal{P}_6\mathcal{F}_6(\chi) &= \frac{1}{(4\pi)^2} (\text{Tr}(\mathbf{V}_\mu \mathbf{V}^\mu))^2 \\
\mathcal{P}_7\mathcal{F}_7(\chi) &= \frac{1}{(4\pi)^2 \chi} \text{Tr}(\mathbf{V}_\mu \mathbf{V}^\mu) \partial_\nu \partial^\nu \chi \\
&\rightarrow \frac{1}{4\pi\Lambda} \text{Tr}(\mathbf{V}_\mu \mathbf{V}^\mu) \partial_\nu \partial^\nu \bar{\chi} \left(1 - \frac{\bar{\chi}}{f} + \frac{\bar{\chi}^2}{f^2} + \dots\right) \\
\mathcal{P}_8\mathcal{F}_8(\chi) &= \frac{1}{(4\pi)^2 \chi^2} \text{Tr}(\mathbf{V}_\mu \mathbf{V}_\nu) \partial^\mu \chi \partial^\nu \chi \\
&\rightarrow \frac{1}{\Lambda^2} \text{Tr}(\mathbf{V}_\mu \mathbf{V}_\nu) \partial^\mu \bar{\chi} \partial^\nu \bar{\chi} \left(1 - \frac{2\bar{\chi}}{f} + \frac{3\bar{\chi}^2}{f^2} + \dots\right) \\
\mathcal{P}_9\mathcal{F}_9(\chi) &= \frac{1}{(4\pi)^2} \text{Tr}((\mathcal{D}_\mu \mathbf{V}^\mu)^2) \\
\mathcal{P}_{10}\mathcal{F}_{10}(\chi) &= \frac{1}{(4\pi)^2 \chi} \text{Tr}(\mathbf{V}_\nu \mathcal{D}_\mu \mathbf{V}^\mu) \partial^\nu \chi \\
&\rightarrow \frac{1}{4\pi\Lambda} \text{Tr}(\mathbf{V}_\nu \mathcal{D}_\mu \mathbf{V}^\mu) \partial^\nu \bar{\chi} \left(1 - \frac{\bar{\chi}}{f} + \frac{\bar{\chi}^2}{f^2} + \dots\right) \\
\mathcal{P}_{11}\mathcal{F}_{11}(\chi) &= \frac{1}{(4\pi)^2} (\text{Tr}(\mathbf{V}_\mu \mathbf{V}_\nu))^2 \\
\mathcal{P}_{12}\mathcal{F}_{12}(\chi) &= (\text{Tr}(\mathbf{T}\mathbf{W}_{\mu\nu}))^2 \\
\mathcal{P}_{13}\mathcal{F}_{13}(\chi) &= \frac{i}{4\pi} \text{Tr}(\mathbf{T}\mathbf{W}_{\mu\nu}) \text{Tr}(\mathbf{T}[\mathbf{V}^\mu, \mathbf{V}^\nu]) \\
\mathcal{P}_{14}\mathcal{F}_{14}(\chi) &= \frac{\varepsilon_{\mu\nu\rho\lambda}}{4\pi} \text{Tr}(\mathbf{T}\mathbf{V}^\mu) \text{Tr}(\mathbf{V}^\nu \mathbf{W}^{\rho\lambda}) \\
\mathcal{P}_{15}\mathcal{F}_{15}(\chi) &= \frac{1}{(4\pi)^2} \text{Tr}(\mathbf{T}\mathcal{D}_\mu \mathbf{V}^\mu) \text{Tr}(\mathbf{T}\mathcal{D}_\nu \mathbf{V}^\nu)
\end{aligned}$$

where the functions $\mathcal{F}_i(\chi)$ correspond to the the last factors inside the brackets, when present, otherwise are equal to 1.

In the previous expressions, part of the operators required the insertion of the conformal regulator in order to recover scale invariance: these terms have been written both in terms of χ , to make explicit the scale invariance although the dilaton kinetic terms are not canonical yet,

$$\begin{aligned}
\mathcal{P}_{16}\mathcal{F}_{16}(\chi) &= \frac{1}{(4\pi)^2} \text{Tr}([\mathbf{T}, \mathbf{V}_\nu] \mathcal{D}_\mu \mathbf{V}^\mu) \text{Tr}(\mathbf{T}\mathbf{V}^\nu) \\
\mathcal{P}_{17}\mathcal{F}_{17}(\chi) &= \frac{i}{4\pi\chi} \text{Tr}(\mathbf{T}\mathbf{W}_{\mu\nu}) \text{Tr}(\mathbf{T}\mathbf{V}^\mu) \partial^\nu \chi \\
&\rightarrow \frac{i}{\Lambda} \text{Tr}(\mathbf{T}\mathbf{W}_{\mu\nu}) \text{Tr}(\mathbf{T}\mathbf{V}^\mu) \partial^\nu \bar{\chi} \left(1 - \frac{\bar{\chi}}{f} + \frac{\bar{\chi}^2}{f^2} + \dots\right) \\
\mathcal{P}_{18}\mathcal{F}_{18}(\chi) &= \frac{1}{(4\pi)^2 \chi} \text{Tr}(\mathbf{T}[\mathbf{V}_\mu, \mathbf{V}_\nu]) \text{Tr}(\mathbf{T}\mathbf{V}^\mu) \partial^\nu \chi \\
&\rightarrow \frac{1}{4\pi\Lambda} \text{Tr}(\mathbf{T}[\mathbf{V}_\mu, \mathbf{V}_\nu]) \text{Tr}(\mathbf{T}\mathbf{V}^\mu) \partial^\nu \bar{\chi} \left(1 - \frac{\bar{\chi}}{f} + \frac{\bar{\chi}^2}{f^2} + \dots\right) \\
\mathcal{P}_{19}\mathcal{F}_{19}(\chi) &= \frac{1}{(4\pi)^2 \chi} \text{Tr}(\mathbf{T}\mathcal{D}_\mu \mathbf{V}^\mu) \text{Tr}(\mathbf{T}\mathbf{V}_\nu) \partial^\nu \chi \\
&\rightarrow \frac{1}{4\pi\Lambda} \text{Tr}(\mathbf{T}\mathcal{D}_\mu \mathbf{V}^\mu) \text{Tr}(\mathbf{T}\mathbf{V}_\nu) \partial^\nu \bar{\chi} \left(1 - \frac{\bar{\chi}}{f} + \frac{\bar{\chi}^2}{f^2} + \dots\right) \\
\mathcal{P}_{20}\mathcal{F}_{20}(\chi) &= \frac{1}{(4\pi)^2 \chi^2} \text{Tr}(\mathbf{V}_\mu \mathbf{V}^\mu) \partial_\nu \chi \partial^\nu \chi \\
&\rightarrow \frac{1}{\Lambda^2} \text{Tr}(\mathbf{V}_\mu \mathbf{V}^\mu) \partial_\nu \bar{\chi} \partial^\nu \bar{\chi} \left(1 - \frac{2\bar{\chi}}{f} + \frac{3\bar{\chi}^2}{f^2} + \dots\right) \\
\mathcal{P}_{21}\mathcal{F}_{21}(\chi) &= \frac{1}{(4\pi)^2 \chi^2} (\text{Tr}(\mathbf{T}\mathbf{V}_\mu))^2 \partial_\nu \chi \partial^\nu \chi \\
&\rightarrow \frac{1}{\Lambda^2} (\text{Tr}(\mathbf{T}\mathbf{V}_\mu))^2 \partial_\nu \bar{\chi} \partial^\nu \bar{\chi} \left(1 - \frac{2\bar{\chi}}{f} + \frac{3\bar{\chi}^2}{f^2} + \dots\right) \\
\mathcal{P}_{22}\mathcal{F}_{22}(\chi) &= \frac{1}{(4\pi)^2 \chi^2} \text{Tr}(\mathbf{T}\mathbf{V}_\mu) \text{Tr}(\mathbf{T}\mathbf{V}_\nu) \partial^\mu \chi \partial^\nu \chi \\
&\rightarrow \frac{1}{\Lambda^2} \text{Tr}(\mathbf{T}\mathbf{V}_\mu) \text{Tr}(\mathbf{T}\mathbf{V}_\nu) \partial^\mu \bar{\chi} \partial^\nu \bar{\chi} \left(1 - \frac{2\bar{\chi}}{f} + \frac{3\bar{\chi}^2}{f^2} + \dots\right) \\
\mathcal{P}_{23}\mathcal{F}_{23}(\chi) &= \frac{1}{(4\pi)^2} \text{Tr}(\mathbf{V}_\mu \mathbf{V}^\mu) (\text{Tr}(\mathbf{T}\mathbf{V}_\nu))^2 \\
\mathcal{P}_{24}\mathcal{F}_{24}(\chi) &= \frac{1}{(4\pi)^2} \text{Tr}(\mathbf{V}_\mu \mathbf{V}_\nu) \text{Tr}(\mathbf{T}\mathbf{V}^\mu) \text{Tr}(\mathbf{T}\mathbf{V}^\nu) \\
\mathcal{P}_{25}\mathcal{F}_{25}(\chi) &= \frac{1}{(4\pi)^2 \chi} (\text{Tr}(\mathbf{T}\mathbf{V}_\mu))^2 \partial_\nu \partial^\nu \chi \\
&\rightarrow \frac{1}{4\pi\Lambda} (\text{Tr}(\mathbf{T}\mathbf{V}_\mu))^2 \partial_\nu \partial^\nu \bar{\chi} \left(1 - \frac{\bar{\chi}}{f} + \frac{\bar{\chi}^2}{f^2} + \dots\right) \\
\mathcal{P}_{26}\mathcal{F}_{26}(\chi) &= \frac{1}{(4\pi)^2} (\text{Tr}(\mathbf{T}\mathbf{V}_\mu) \text{Tr}(\mathbf{T}\mathbf{V}_\nu))^2,
\end{aligned} \tag{37}$$

and in terms of $\bar{\chi}$, that is in the canonical kinetic term basis. The rest of operators are naturally of canonical dimension $\mathbf{d} = 4$ and thus scale invariant by themselves.

As for the HEFT operators listed in Eq. (18), those containing the scalar chiral field \mathbf{T} , not in association to the gauge field strength $B_{\mu\nu}$, represent sources of custodial symmetry breaking beyond those of the SM: specifically, they are $\mathcal{P}_{12} - \mathcal{P}_{26}$, except for \mathcal{P}_{20} .

Eq. (37) completes the DEFT operator basis describing CP-even bosonic couplings of the SM EW sector and the Higgs-like dilaton.

IV. DISENTANGLING A HIGGS-LIKE DILATON

A Higgs-like dilaton is typically considered to have the same couplings as the SM Higgs, exception made for the self non-derivative couplings entering the scalar potential. The previous section illustrates that this is the case for a restrict group of operators, while there are several other couplings which differ from the SM context. This section is dedicated to discuss these differences considering the SMEFT Lagrangian, whose specific limit is the SM, and the minimal $SO(5)/SO(4)$ CH model [27], making use of the HEFT Lagrangian defined in Sect. II as a common background. The explicit operators basis for the SMEFT Lagrangian can be found in App. A, while the one for the minimal $SO(5)/SO(4)$ CH model in App. B.

The first coupling that can be considered for this comparison is the one responsible for the W^\pm and Z masses. In the SMEFT up to $d = 6$ expansion order, there are four relevant operators:

$$\mathbf{D}_\mu \Phi^\dagger \mathbf{D}^\mu \Phi + \frac{c_{\Phi,1}}{\Lambda^2} \mathcal{O}_{\Phi,1} + \frac{c_{\Phi,2}}{\Lambda^2} \mathcal{O}_{\Phi,2} + \frac{c_{\Phi,4}}{\Lambda^2} \mathcal{O}_{\Phi,4}, \quad (38)$$

where the explicit definition of the operators $\mathcal{O}_{\Phi,i}$ can be found in Eq. (A3). As described in App. A, $\mathcal{O}_{\Phi,1}$ and $\mathcal{O}_{\Phi,2}$ contribute to the EW gauge boson masses only indirectly, through the transformation to move to the basis of canonical Higgs kinetic terms. In the latter basis, projecting into the HEFT basis, $\mathcal{F}_C(h)$ is given by:

$$\begin{aligned} \mathcal{F}_C^{\text{SMEFT}}(h) = & \left[\left(1 + \frac{(4\pi)^2 v^2}{2 \Lambda^2} c_{\Phi 4} \right) + \right. \\ & + 2 \frac{h}{v} \left(1 - \frac{(4\pi)^2 v^2}{4 \Lambda^2} (c_{\Phi 1} + 2c_{\Phi 2} - 3c_{\Phi 4}) \right) \\ & + \frac{h^2}{v^2} \left(1 - (4\pi)^2 \frac{v^2}{\Lambda^2} (c_{\Phi 1} + 2c_{\Phi 2} - 2c_{\Phi 4}) \right) \\ & \left. + \dots \right], \end{aligned} \quad (39)$$

where the dots stand for terms proportional to h^3 and h^4 . When the coefficients $c_{\Phi,i} = 0$, then the SM case is recovered.

For the minimal $SO(5)/SO(4)$ CH model and considering up to the NLO expansion order, only one contribution is present:

$$-\frac{f^2}{4} \text{Tr} \left(\tilde{\mathbf{V}}_\mu \tilde{\mathbf{V}}^\mu \right) = \frac{v^2}{4} \text{Tr}(\mathbf{V}_\mu \mathbf{V}^\mu) \left(\frac{4f^2}{v^2} \sin^2 \frac{\varphi}{2f} \right), \quad (40)$$

where $\tilde{\mathbf{V}}$ is the generalised vector chiral field in the $SO(5)$ representation, sibling of the vector chiral field

\mathbf{V} of the EW chiral Lagrangian, and $\varphi = h + \langle \varphi \rangle$, being $\sin^2(\langle \varphi \rangle / 2f) = v^2 / 4f^2$. The $\mathcal{F}_C(h)$ function for this case can easily be read from the previous expression:

$$\mathcal{F}_C^{\text{CH}}(h) = \frac{4f^2}{v^2} \sin^2 \frac{h + \langle \varphi \rangle}{2f}. \quad (41)$$

These results should be compared with the equivalent contributions in the dilaton context, Eqs. (27) and (31b): using a similar notation as for the HEFT, one can write

$$\mathcal{P}_C \mathcal{F}_C(\chi) = \frac{v^2}{4} \text{Tr}(\mathbf{V}_\mu \mathbf{V}^\mu) \left(1 + \frac{\bar{\chi}}{f} \right)^2, \quad (42)$$

which leads to

$$\mathcal{F}_C^{\text{DEFT}}(\chi) = \left(1 + \frac{\bar{\chi}}{f} \right)^2. \quad (43)$$

All the three descriptions expect deviations from the SM predictions for the Higgs couplings to longitudinal components of the massive gauge bosons, and they differ from each other. The DEFT is the only one presenting up to quadratic powers of h and with the same numerical factors as in the SM, even if with a v/f suppressing factor (see also Refs. [32, 37]).

A second group of relevant couplings that deserves a dedicated discussion is represented by the Yukawa interactions. In general, deviations from the SM predictions are expected in all the frameworks. Here, however, the focus is on the bosonic Lagrangians only and no genuine fermionic operator is considered. In consequence, the only modification to the SM Yukawa interactions that can arise is due to the transformation to move to the basis of canonical kinetic terms, which in particular occurs in both the SMEFT and the DEFT Lagrangians. Then, the functions multiplying the fermionic bilinear $(v/\sqrt{2})\bar{\psi}_L \mathbf{U} Y_\psi^{(0)} \psi_R$ read:

$$\begin{aligned} \mathcal{F}_{U,D,\ell}^{\text{SMEFT}}(h) = & 1 + \frac{h}{v} \left(1 - \frac{(4\pi)^2}{12} (c_{\Phi 1} + 2c_{\Phi 2} + c_{\Phi 4}) \times \right. \\ & \left. \times \frac{3v^2 + 3vh + h^2}{\Lambda^2} \right) \\ \mathcal{F}_{U,D,\ell}^{\text{CH}}(h) = & \sin \frac{h + \langle \varphi \rangle}{f} \\ \mathcal{F}_{U,D,\ell}^{\text{DEFT}}(\chi) = & 1 + \frac{\bar{\chi}}{f}. \end{aligned} \quad (44)$$

For the minimal CH model, the $\mathcal{F}_{U,D,\ell}^{\text{CH}}(h)$ expression follows the result of the original formulation in Ref. [27]. As for the couplings with the longitudinal components of the EW gauge bosons, only the DEFT setup presents the same powers of h and numerical factors as in the SM, although the v/f factor.

The rest of couplings and the comparison among the different contributions from SMEFT, minimal

$c_i \mathcal{F}_i(h)$	SMEFT $\mathbf{d} \leq 6$	$SO(5)/SO(4)$	$d_i \mathcal{F}_i(\chi)$	DEFT
$c_B \mathcal{F}_B(h)$	$2(4\pi)^2 \frac{v^2}{\Lambda^2} c_{BB} \left(1 + \frac{h}{v}\right)^2$	$-4\tilde{c}_{B\Sigma} \cos^2 \frac{\varphi}{2f}$	$d_B \mathcal{F}_B(\chi)$	$d_B \frac{\bar{\chi}}{f}$
$c_W \mathcal{F}_W(h)$	$\frac{(4\pi)^2}{2} \frac{v^2}{\Lambda^2} c_{WW} \left(1 + \frac{h}{v}\right)^2$	$-4\tilde{c}_{W\Sigma} \cos^2 \frac{\varphi}{2f}$	$d_W \mathcal{F}_W(\chi)$	$d_W \frac{\bar{\chi}}{f}$
$c_G \mathcal{F}_G(h)$	$2(4\pi)^2 \frac{v^2}{\Lambda^2} c_{GG} \left(1 + \frac{h}{v}\right)^2$	*	$d_G \mathcal{F}_G(\chi)$	$d_G \frac{\bar{\chi}}{f}$
$c_{\square H} \mathcal{F}_{\square H}(h)$	$\frac{1}{2} c_{\square\Phi}$	$-2\tilde{c}_6$	$d_{\square\chi} \mathcal{F}_{\square\chi}(\chi)$	$d_{\square\chi} \left(1 - \frac{2\bar{\chi}}{f} + \frac{3\bar{\chi}^2}{f^2} + \dots\right)$
$c_{\Delta H} \mathcal{F}_{\Delta H}(h)$	—	—	$d_{\Delta\chi} \mathcal{F}_{\Delta\chi}(\chi)$	$d_{\Delta\chi} \left(1 - \frac{3\bar{\chi}}{f} + \frac{6\bar{\chi}^2}{f^2} + \dots\right)$
$c_{DH} \mathcal{F}_{DH}(h)$	—	$4(\tilde{c}_4 + \tilde{c}_5)$	$d_{D\chi} \mathcal{F}_{D\chi}(\chi)$	$d_{D\chi} \left(1 - \frac{4\bar{\chi}}{f} + \frac{10\bar{\chi}^2}{f^2} + \dots\right)$
$c_{WWW} \mathcal{F}_{WWW}(h)$	c_{3W}	—	$d_{WWW} \mathcal{F}_{WWW}(\chi)$	$d_{WWW} \left(1 - \frac{2\bar{\chi}}{f} + \frac{3\bar{\chi}^2}{f^2} + \dots\right)$
$c_{GGG} \mathcal{F}_{GGG}(h)$	c_{3G}	—	$d_{GGG} \mathcal{F}_{GGG}(\chi)$	$d_{GGG} \left(1 - \frac{2\bar{\chi}}{f} + \frac{3\bar{\chi}^2}{f^2} + \dots\right)$
$c_{DB} \mathcal{F}_{DB}(h)$	c_{DB}	—	$d_{DB} \mathcal{F}_{DB}(\chi)$	$d_{DB} \left(1 - \frac{2\bar{\chi}}{f} + \frac{3\bar{\chi}^2}{f^2} + \dots\right)$
$c_{DW} \mathcal{F}_{DW}(h)$	c_{DW}	—	$d_{DW} \mathcal{F}_{DW}(\chi)$	$d_{DW} \left(1 - \frac{2\bar{\chi}}{f} + \frac{3\bar{\chi}^2}{f^2} + \dots\right)$
$c_{DG} \mathcal{F}_{DG}(h)$	c_{DG}	—	$d_{DG} \mathcal{F}_{DG}(\chi)$	$d_{DG} \left(1 - \frac{2\bar{\chi}}{f} + \frac{3\bar{\chi}^2}{f^2} + \dots\right)$
$c_1 \mathcal{F}_1(h)$	$\frac{(4\pi)^2}{8} \frac{v^2}{\Lambda^2} c_{BW} \left(1 + \frac{h}{v}\right)^2$	$\tilde{c}_1 \sin^2 \frac{\varphi}{2f}$	$d_1 \mathcal{F}_1(\chi)$	$d_1 \frac{\bar{\chi}}{f}$
$c_2 \mathcal{F}_2(h)$	$\frac{(4\pi)^2}{8} \frac{v^2}{\Lambda^2} c_B \left(1 + \frac{h}{v}\right)^2$	$\tilde{c}_2 \sin^2 \frac{\varphi}{2f}$	$d_2 \mathcal{F}_2(\chi)$	d_2
$c_3 \mathcal{F}_3(h)$	$\frac{(4\pi)^2}{4} \frac{v^2}{\Lambda^2} c_W \left(1 + \frac{h}{v}\right)^2$	$2\tilde{c}_3 \sin^2 \frac{\varphi}{2f}$	$d_3 \mathcal{F}_3(\chi)$	d_3
$c_4 \mathcal{F}_4(h)$	$\frac{4\pi}{2} \frac{v}{\Lambda} c_B \left(1 + \frac{h}{v}\right)$	$\tilde{c}_2 \sin \frac{\varphi}{f}$	$d_4 \mathcal{F}_4(\chi)$	$d_4 \left(1 - \frac{\bar{\chi}}{f} + \frac{\bar{\chi}^2}{f^2} + \dots\right)$
$c_5 \mathcal{F}_5(h)$	$-4\pi \frac{v}{\Lambda} c_W \left(1 + \frac{h}{v}\right)$	$-2\tilde{c}_3 \sin \frac{\varphi}{f}$	$d_5 \mathcal{F}_5(\chi)$	$d_5 \left(1 - \frac{\bar{\chi}}{f} + \frac{\bar{\chi}^2}{f^2} + \dots\right)$
$c_6 \mathcal{F}_6(h)$	$\frac{(4\pi)^2}{8} \frac{v^2}{\Lambda^2} c_{\square\Phi} \left(1 + \frac{h}{v}\right)^2$	$16\tilde{c}_4 \sin^4 \frac{\varphi}{2f} - \frac{1}{2}\tilde{c}_6 \sin^2 \frac{\varphi}{f}$	$d_6 \mathcal{F}_6(\chi)$	d_6
$c_7 \mathcal{F}_7(h)$	$\frac{4\pi}{2} \frac{v}{\Lambda} c_{\square\Phi} \left(1 + \frac{h}{v}\right)$	$-2\tilde{c}_6 \sin \frac{\varphi}{f}$	$d_7 \mathcal{F}_7(\chi)$	$d_7 \left(1 - \frac{\bar{\chi}}{f} + \frac{\bar{\chi}^2}{f^2} + \dots\right)$
$c_8 \mathcal{F}_8(h)$	$-c_{\square\Phi}$	$-16\tilde{c}_5 \sin^2 \frac{\varphi}{2f} + 4\tilde{c}_6 \cos^2 \frac{\varphi}{2f}$	$d_8 \mathcal{F}_8(\chi)$	$d_8 \left(1 - \frac{2\bar{\chi}}{f} + \frac{3\bar{\chi}^2}{f^2} + \dots\right)$
$c_9 \mathcal{F}_9(h)$	$-\frac{(4\pi)^2}{4} \frac{v^2}{\Lambda^2} c_{\square\Phi} \left(1 + \frac{h}{v}\right)^2$	$4\tilde{c}_6 \sin^2 \frac{\varphi}{2f}$	$d_9 \mathcal{F}_9(\chi)$	d_9
$c_{10} \mathcal{F}_{10}(h)$	$-4\pi \frac{v}{\Lambda} c_{\square\Phi} \left(1 + \frac{h}{v}\right)$	$4\tilde{c}_6 \sin \frac{\varphi}{f}$	$d_{10} \mathcal{F}_{10}(\chi)$	$d_{10} \left(1 - \frac{\bar{\chi}}{f} + \frac{\bar{\chi}^2}{f^2} + \dots\right)$
$c_{11} \mathcal{F}_{11}(h)$	—	$16\tilde{c}_5 \sin^4 \frac{\varphi}{2f}$	$d_{11} \mathcal{F}_{11}(\chi)$	d_{11}
$c_{20} \mathcal{F}_{20}(h)$	—	$-16\tilde{c}_4 \sin^2 \frac{\varphi}{2f}$	$d_{20} \mathcal{F}_{20}(\chi)$	$d_{20} \left(1 - \frac{2\bar{\chi}}{f} + \frac{3\bar{\chi}^2}{f^2} + \dots\right)$

TABLE I. Explicit expressions for the products $c_i \mathcal{F}_i(h)$ and $d_i \mathcal{F}_i(h)$ of the low-energy custodial preserving operators. The “—” entries indicate the absence of contributions at the order considered to the corresponding low-energy operator. The “*” indicates that couplings to gluons in CH models are expected due to fermionic loops (see App. B).

$SO(5)/SO(4)$ CH model and DEFT are summarised in Tab. I, restricting to the only custodial preserving contributions.

The custodial breaking couplings will be reported in Tab. II only for the DEFT basis. Indeed, the minimal $SO(5)/SO(4)$ CH model does not allow these couplings to appear, and for the SMEFT basis only the operator $\mathcal{O}_{\Phi,1}$ is custodial breaking: projecting it into the HEFT

basis, one finds that the function $\mathcal{F}_T(h)$ receives the following contribution,

$$c_T \mathcal{F}_T^{\text{SMEFT}}(h) = -\frac{(4\pi)^2}{4} \frac{v^2}{\Lambda^2} c_T \left(1 + \frac{h}{v}\right)^2. \quad (45)$$

This can be compared with the corresponding contribu-

$d_i \mathcal{F}_i(\chi)$	DEFT	$d_i \mathcal{F}_i(\chi)$	DEFT	$d_i \mathcal{F}_i(\chi)$	DEFT
$d_{12} \mathcal{F}_{12}(\chi)$	d_{12}	$d_{17} \mathcal{F}_{17}(\chi)$	$d_{17} \left(1 - \frac{\bar{\chi}}{f} + \frac{\bar{\chi}^2}{f^2} + \dots\right)$	$d_{23} \mathcal{F}_{23}(\chi)$	d_{23}
$d_{13} \mathcal{F}_{13}(\chi)$	d_{13}	$d_{18} \mathcal{F}_{18}(\chi)$	$d_{18} \left(1 - \frac{\bar{\chi}}{f} + \frac{\bar{\chi}^2}{f^2} + \dots\right)$	$d_{24} \mathcal{F}_{24}(\chi)$	d_{24}
$d_{14} \mathcal{F}_{14}(\chi)$	d_{14}	$d_{19} \mathcal{F}_{19}(\chi)$	$d_{19} \left(1 - \frac{\bar{\chi}}{f} + \frac{\bar{\chi}^2}{f^2} + \dots\right)$	$d_{25} \mathcal{F}_{25}(\chi)$	$d_{25} \left(1 - \frac{\bar{\chi}}{f} + \frac{\bar{\chi}^2}{f^2} + \dots\right)$
$d_{15} \mathcal{F}_{15}(\chi)$	d_{15}	$d_{21} \mathcal{F}_{21}(\chi)$	$d_{21} \left(1 - \frac{2\bar{\chi}}{f} + \frac{3\bar{\chi}^2}{f^2} + \dots\right)$	$d_{26} \mathcal{F}_{26}(\chi)$	d_{26}
$d_{16} \mathcal{F}_{16}(\chi)$	d_{16}	$d_{22} \mathcal{F}_{22}(\chi)$	$d_{22} \left(1 - \frac{2\bar{\chi}}{f} + \frac{3\bar{\chi}^2}{f^2} + \dots\right)$		

TABLE II. Explicit expressions for the products $d_i \mathcal{F}_i(h)$ of the low-energy custodial breaking operators.

tion in the DEFT basis:

$$d_T \mathcal{F}_T^{\text{DEFT}}(\chi) = d_T \left(1 + \frac{\bar{\chi}}{f}\right)^2. \quad (46)$$

A. Phenomenology Avenues

As previously discussed in Eqs. (43), (44) and (46) for the lowest order Lagrangian, the Higgs-like dilaton has couplings much similar to the ones of the SM Higgs, at least for the shape of these couplings: a tuning of v/f is necessary to match the correct order of magnitude of the dilaton interactions with longitudinal components of gauge bosons and with fermions; besides this, the numerical value of the correlation between 0, 1 and 2 dilaton insertions in a specific coupling is the same as in the SM.

Going beyond the lowest order Lagrangian, Tabs. I and II allow to discuss signals that could be able to disentangle a Higgs-like dilaton from other possibilities. The features that jump to the eyes are:

- i) the specific linear combinations of h and φ in the $\mathcal{F}(h)$ functions and of $\bar{\chi}$ in the $\mathcal{F}_i(\chi)$ functions;
- ii) the presence of a few pure-gauge couplings that do not come along with $\bar{\chi}$ insertions, while these same couplings appear with h insertions in the SMEFT and in the CH setup: \mathcal{P}_2 , \mathcal{P}_3 , \mathcal{P}_6 , \mathcal{P}_9 , \mathcal{P}_{11-16} , \mathcal{P}_{23} , \mathcal{P}_{24} and \mathcal{P}_{26} ;
- iii) the presence of $\bar{\chi}$ insertions couplings, that are instead predicted to be only pure-gauge in the SMEFT and in the CH framework: $\mathcal{P}_{\square\chi}$, $\mathcal{P}_{\Delta\chi}$, $\mathcal{P}_{D\chi}$, \mathcal{P}_{WWW} , \mathcal{P}_{GGG} , \mathcal{P}_{DB} , \mathcal{P}_{DW} and \mathcal{P}_{DG} ;
- iv) the independence of the couplings described in each line of the tables due to the d_i coefficients, while SMEFT and CH Lagrangian predict correlations between couplings of different lines: for example, $\mathcal{F}_2(h)$ and $\mathcal{F}_4(h)$, or $\mathcal{F}_3(h)$ and $\mathcal{F}_5(h)$, etc..
- v) all the custodial breaking terms in Tab. II are not expected neither in the $d = 6$ SMEFT Lagrangian, nor in the minimal $SO(5)/SO(4)$ CH model.

For what concerns the point i), the specific correlations between 0, 1, 2, ..., dilaton insertion in a given coupling are distinct from those in the SMEFT and in CH models: this could indeed discriminate between the different frameworks, once the sensitivity on Higgs couplings will increase. Several studies on phenomenological signals of a Higgs-like dilaton at colliders have been presented, see for example Refs. [32, 37].

The absence of dilaton couplings reported in point ii) is connected to another type of potentially interesting phenomenological signals. For example, the operator \mathcal{P}_2 describes only $\gamma - W - W$ and $Z - W - W$ pure-gauge couplings with specific Lorentz contractions, and in particular no dilaton insertion is expected in the DEFT Lagrangian; in the SMEFT and CH setups, these pure-gauge couplings arise due to the \mathcal{P}_2 operator and are predicted to be correlated to the Higgs couplings $\gamma - W - W - h$ and $Z - W - W - h$, respectively. In a one-operator-at-a-time analysis, the absence of the dilaton insertion is a smoking gun for this scenario; on the other side, in a multi-operatorial analysis, the discriminating power of this feature is much reduced and the impact of this operator should be analysed together with the effects of the other Lagrangian operators.

The case described in point iii) is the opposite with respect to what just discussed about point ii): couplings with dilaton insertions, whose corresponding couplings with h are not expected in the SMEFT and CH setup, at the expanding order considered. The discussion and conclusions of the previous paragraph also apply to this case, after the necessary rephrasing.

Point iv) states that several couplings that are expected to be correlated (at a given uncertainty) in the SMEFT and CH framework, are instead decorrelated in the DEFT case. This is the case, for example, of \mathcal{P}_2 and \mathcal{P}_4 , or \mathcal{P}_3 and \mathcal{P}_5 . The kind of the decorrelation of the DEFT is the same of the generic HEFT Lagrangian discussed in Ref. [14, 17]: see in particular Fig. 3 of Ref. [17], that also applies to the DEFT Lagrangian, after redefin-

ing the Σ and Δ functions:

$$\begin{aligned}\Sigma_B &\equiv \frac{1}{\pi g t_\theta} (2d_2 - d_4), & \Sigma_W &\equiv \frac{1}{2\pi g} (2d_3 + d_5), \\ \Delta_B &\equiv \frac{1}{\pi g t_\theta} (2d_2 + d_4), & \Delta_W &\equiv \frac{1}{2\pi g} (2d_3 - d_5),\end{aligned}\quad (47)$$

where t_θ is the tangent of the Weinberg angle. The two Δ 's parameters are zero in the $\mathbf{d} = 6$ SMEFT Lagrangian in consequence of gauge invariance and the doublet nature of the Higgs. $\Delta_B = 0 = \Delta_W$ also in the minimal $SO(5)/SO(4)$ CH model once v^4/f^4 terms are neglected, due to the almost exactly doublet embedding of the Higgs. Then, deviations from zero larger than v^4/f^4 cannot be explained with $\mathbf{d} = 8$ SMEFT contributions or with contributions from the $SO(5)/SO(4)$ CH Lagrangian, while they could be compatible with the DEFT description. The orthogonal combinations, Σ_B and Σ_W will instead represent deviations from the exact SM case and cannot distinguish between $\mathbf{d} = 6$ SMEFT, $SO(5)/SO(4)$ CH or DEFT contributions. Further details can be found in Ref. [17].

Finally, the custodial breaking contributions in point v) are only present in the DEFT Lagrangian, while are not expected neither in the $\mathbf{d} = 6$ SMEFT Lagrangian, nor in the minimal $SO(5)/SO(4)$ CH model. They represent interesting possibilities to tell the dilaton from the other frameworks: this is the case of the pure-gauge operator \mathcal{P}_{14} , equal to \mathcal{P}_{14} , discussed in Ref. [14]; this operator contributes to the anomalous triple gauge vertex

$$ig_Z^5 \epsilon^{\mu\nu\rho\lambda} (W_\mu^+ \partial_\rho W_\nu^- - W_\nu^- \partial_\rho W_\mu^+) Z_\sigma, \quad (48)$$

that originates WW and WZ pair production. The second observable has been studied in Ref. [14] analysing the reaction

$$pp \rightarrow \ell'^{\pm} \ell^+ \ell^- E_T^{\text{miss}}, \quad (49)$$

where $\ell^{(\prime)} = e$ or μ . The predicted values of the cross sections for this process considering 7 TeV, 8 TeV, and 14 TeV center of mass energy have been computed and the expected number of events over the background has been illustrated in Fig. 3 of Ref. [14]. The results are that the precision on g_Z^5 at the LHC 7 and 8 TeV runs is already higher than the present direct bounds stemming from LEP and therefore LHC has already the potential to discriminate from a Higgs-like dilaton from an (almost) exactly EW doublet Higgs.

V. CONCLUSIONS

The complete bosonic effective Lagrangian for a Higgs-like dilaton is constructed considering an expansion up to the first subleading order. This basis is compared with the $\mathbf{d} = 6$ SMEFT Lagrangian and with the effective Lagrangian for the minimal $SO(5)/SO(4)$ CH model.

Five distinct features that could disentangle between the different Higgs descriptions are discussed: they are

related to the presence (or absence) of specific correlations among dilaton insertions, which are distinct from the correlations among h insertions in SMEFT and CH setups; moreover, they are due to the larger number of independent operators in the DEFT basis; finally, they follow the fact that custodial breaking contributions are only predicted in the DEFT Lagrangian, at the considered expansion order.

Previous studies on the generic HEFT Lagrangian also hold for the DEFT Lagrangian: correlations/decorrelations between pure-gauge couplings and dilaton-gauge bosons interactions; anomalous triple gauge boson couplings only predicted to be relevant in the DEFT case. They represent promising signals to disentangle a Higgs-like dilaton from an (almost) exact EW doublet Higgs at colliders.

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Appendix A: The SMEFT Lagrangian

The SMEFT Lagrangian [4, 5] is constructed by means of the SM particles and in particular with the Higgs field belonging to a doublet of the $SU(2)_L$ symmetry, Eq. (1). When restricting only on the bosonic sector and only on operators of $\mathbf{d} \leq 6$, the list of terms defining a non-redundant basis counts 11 elements describing Higgs interactions – the so-called Hagiwara-Ishihara-Szalapski-Zeppenfeld (HISZ) basis [92, 93] – plus additional 5 elements containing pure-gauge couplings [94]. Considering these restrictions, the Lagrangian can be written as a sum of two terms,

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \Delta \mathcal{L}_{\text{SMEFT}}, \quad (A1)$$

where \mathcal{L}_{SM} is the SM Lagrangian and

$$\Delta \mathcal{L}_{\text{SMEFT}} = \sum_i \frac{c_i}{\Lambda^2} \mathcal{O}_i, \quad (A2)$$

with c_i being order one parameters and \mathcal{O}_i defined as follows:

$$\begin{aligned}
\mathcal{O}_{BB} &= (4\pi)^2 \Phi^\dagger B_{\mu\nu} B^{\mu\nu} \Phi \\
\mathcal{O}_{WW} &= (4\pi)^2 \Phi^\dagger \mathbf{W}_{\mu\nu} \mathbf{W}^{\mu\nu} \Phi \\
\mathcal{O}_{GG} &= (4\pi)^2 \Phi^\dagger \Phi G_{\mu\nu}^\alpha G^{\alpha\mu\nu} \\
\mathcal{O}_{BW} &= (4\pi)^2 \Phi^\dagger B_{\mu\nu} \mathbf{W}^{\mu\nu} \Phi \\
\mathcal{O}_W &= 4\pi i (\mathbf{D}_\mu \Phi)^\dagger \mathbf{W}^{\mu\nu} (\mathbf{D}_\nu \Phi) \\
\mathcal{O}_B &= 4\pi i (\mathbf{D}_\mu \Phi)^\dagger B^{\mu\nu} (\mathbf{D}_\nu \Phi) \\
\mathcal{O}_{\Phi,1} &= (4\pi)^2 (\mathbf{D}_\mu \Phi)^\dagger \Phi \Phi^\dagger (\mathbf{D}^\mu \Phi) \\
\mathcal{O}_{\Phi,2} &= (4\pi)^2 \frac{1}{2} \partial^\mu (\Phi^\dagger \Phi) \partial_\mu (\Phi^\dagger \Phi) \\
\mathcal{O}_{\Phi,3} &= (4\pi)^4 (\Phi^\dagger \Phi)^3 \\
\mathcal{O}_{\Phi,4} &= (4\pi)^2 (\mathbf{D}_\mu \Phi)^\dagger (\mathbf{D}^\mu \Phi) (\Phi^\dagger \Phi) \\
\mathcal{O}_{\square\Phi} &= (\mathbf{D}_\mu \mathbf{D}^\mu \Phi)^\dagger (\mathbf{D}_\nu \mathbf{D}^\nu \Phi) \\
\mathcal{O}_{3W} &= 4\pi \varepsilon_{abc} W_\mu^{a\nu} W_\nu^{b\rho} W_\rho^{c\mu} \\
\mathcal{O}_{3G} &= 4\pi f_{\alpha\beta\gamma} G_\mu^{\alpha\nu} G_\nu^{\beta\rho} G_\rho^{\gamma\mu} \\
\mathcal{P}_{DB} &= (\partial^\mu B_{\mu\nu}) (\partial_\rho B^{\rho\nu}) \\
\mathcal{P}_{DW} &= (\mathcal{D}^\mu W_{\mu\nu})^a (\mathcal{D}_\rho W^{\rho\nu})^a \\
\mathcal{P}_{DG} &= (\mathcal{D}^\mu G_{\mu\nu})^\alpha (\mathcal{D}_\rho G^{\rho\nu})^\alpha.
\end{aligned} \tag{A3}$$

In the previous expressions, $\mathbf{D}_\mu \Phi \equiv (\partial_\mu + \frac{i}{2} g' B_\mu + \frac{i}{2} g \sigma_i W_\mu^i) \Phi$ is the covariant derivative applied to the Higgs doublet. Among these operators, only $\mathcal{O}_{\Phi,1}$ is custodial breaking, while the rest are custodial preserving. All the operators are written according to the Naive Dimensional Analysis normalisation [22].

The operators $\mathcal{O}_{\Phi,1}$, $\mathcal{O}_{\Phi,2}$ and $\mathcal{O}_{\Phi,4}$ contribute to the Higgs kinetic term:

$$\frac{1}{2} \partial_\mu h \partial^\mu h \left(1 + \frac{(4\pi)^2}{2} \frac{(v+h)^2}{\Lambda^2} (c_{\Phi 1} + 2c_{\Phi 2} + c_{\Phi 4}) \right). \tag{A4}$$

It is then necessary to perform a field redefinition to move to the basis of canonical kinetic terms (see App. B in Ref. [17]): neglecting terms proportional to products of two or more c_i , the transformation reduces to

$$h \rightarrow h \left(1 - \frac{(4\pi)^2}{12} (c_{\Phi 1} + 2c_{\Phi 2} + c_{\Phi 4}) \frac{3v^2 + 3vh + h^2}{\Lambda^2} \right). \tag{A5}$$

A priori, all the Higgs couplings in the full Lagrangian would be affected by this transformation; however, neglecting all the terms proportional to products of two or more c_i , only the Higgs interactions present in the SM Lagrangian will be modified, i.e. the couplings with two longitudinal components of the EW gauge bosons and the Yukawa terms.

Appendix B: The minimal $SO(5)/SO(4)$ CH Lagrangian

The Lagrangian for the $SO(5)/SO(4)$ minimal CH model is known already from some time [27, 29, 30, 38, 39], and the notation used in Ref. [38, 39] will be adopted in what follows. Mimicking the Appelquist, Longhitano and Feruglio convention, one can define a field matrix $\Omega(x)$ containing all the GBs originated from the $SO(5)/SO(4)$ spontaneous breaking and transforming under the global groups $SO(5)$ and $SO(4)$ as

$$\Omega(x) \rightarrow \mathfrak{g} \Omega(x) \mathfrak{h}^{-1}, \tag{B1}$$

being \mathfrak{g} an element of $SO(5)$ and \mathfrak{h} an element of $SO(4)$. It is then possible to define a “squared” non-linear field matrix $\Sigma(x)$ transforming only under $SO(5)$:

$$\Sigma(x) \equiv \Omega(x)^2, \quad \Sigma(x) \rightarrow \mathfrak{g} \Sigma(x) \mathfrak{g}_R^{-1}, \tag{B2}$$

where \mathfrak{g}_R is the grading of \mathfrak{g} (see Ref. [38, 39] for further details). It is now possible to define the quantity $\tilde{\mathbf{V}}$, that is equivalent to the vector chiral field \mathbf{V} of the EW chiral Lagrangian:

$$\tilde{\mathbf{V}}_\mu = (\mathbf{D}_\mu \Sigma) \Sigma^{-1}, \quad \tilde{\mathbf{V}}_\mu \rightarrow \mathfrak{g} \tilde{\mathbf{V}}_\mu \mathfrak{g}^{-1}, \tag{B3}$$

where \mathbf{D}_μ is the covariant derivative acting on Σ .

Next, one can consider the gauging of the SM group, $SU(2)_L \times U(1)_Y \subset SO(5)$ and $SU(2)_L \times U(1)_Y \not\subset SO(4)$, and the embedding of the SM gauge vector boson into the $SO(5)$ representations,

$$\tilde{\mathbf{W}}_\mu \equiv W_\mu^a Q_L^a \quad \text{and} \quad \tilde{\mathbf{B}}_\mu \equiv B_\mu Q_Y, \tag{B4}$$

where Q_L^a and Q_Y denote the embedding in $SO(5)$ of the $SU(2)_L \times U(1)_Y$ generators.

Finally, the CP-even EW high-energy chiral Lagrangian describing bosonic interactions, \mathcal{L}_{CH} , can be written as:

$$\mathcal{L}_{CH} = \mathcal{L}_{CH}^0 + \Delta \mathcal{L}_{CH}, \tag{B5}$$

where

$$\begin{aligned}
\mathcal{L}_{CH}^0 &= -\frac{1}{4} \text{Tr} \left(\tilde{\mathbf{B}}_{\mu\nu} \tilde{\mathbf{B}}^{\mu\nu} \right) - \frac{1}{4} \text{Tr} \left(\tilde{\mathbf{W}}_{\mu\nu} \tilde{\mathbf{W}}^{\mu\nu} \right) + \\
&\quad -\frac{1}{4} \mathcal{G}_{\mu\nu}^\alpha \mathcal{G}^{\alpha\mu\nu} - \frac{f^2}{4} \text{Tr} \left(\tilde{\mathbf{V}}_\mu \tilde{\mathbf{V}}^\mu \right) + i \bar{Q}_L \not{D} Q_L + \\
&\quad + i \bar{Q}_R \not{D} Q_R + i \bar{L}_L \not{D} L_L + i \bar{L}_R \not{D} L_R + \\
&\quad - \frac{f}{\sqrt{2}} \left(\bar{Q}_L \mathbf{U} Y_Q^{(0)} Q_R \sin \frac{\varphi}{f} + \text{h.c.} \right) + \\
&\quad - \frac{f}{\sqrt{2}} \left(\bar{L}_L \mathbf{U} Y_L^{(0)} L_R \sin \frac{\varphi}{f} + \text{h.c.} \right),
\end{aligned} \tag{B6}$$

and

$$\Delta \mathcal{L}_{CH} = \tilde{c}_{B\Sigma} \tilde{\mathcal{A}}_{B\Sigma} + \tilde{c}_{W\Sigma} \tilde{\mathcal{A}}_{W\Sigma} + \sum_{i=1}^6 \tilde{c}_i \tilde{\mathcal{A}}_i. \tag{B7}$$

The operators appearing in the previous expression are defined by

$$\begin{aligned}
\tilde{\mathcal{A}}_{B\Sigma} &= \text{Tr} \left(\Sigma \tilde{\mathbf{B}}_{\mu\nu} \Sigma^{-1} \tilde{\mathbf{B}}^{\mu\nu} \right) \\
\tilde{\mathcal{A}}_{W\Sigma} &= \text{Tr} \left(\Sigma \tilde{\mathbf{W}}_{\mu\nu} \Sigma^{-1} \tilde{\mathbf{W}}^{\mu\nu} \right) \\
\tilde{\mathcal{A}}_1 &= \text{Tr} \left(\Sigma \tilde{\mathbf{B}}_{\mu\nu} \Sigma^{-1} \tilde{\mathbf{W}}^{\mu\nu} \right) \\
\tilde{\mathcal{A}}_2 &= \frac{i}{4\pi} \text{Tr} \left(\tilde{\mathbf{B}}_{\mu\nu} \left[\tilde{\mathbf{V}}^\mu, \tilde{\mathbf{V}}^\nu \right] \right) \\
\tilde{\mathcal{A}}_3 &= \frac{i}{4\pi} \text{Tr} \left(\tilde{\mathbf{W}}_{\mu\nu} \left[\tilde{\mathbf{V}}^\mu, \tilde{\mathbf{V}}^\nu \right] \right) \\
\tilde{\mathcal{A}}_4 &= \frac{1}{(4\pi)^2} \left(\text{Tr} \left(\tilde{\mathbf{V}}_\mu \tilde{\mathbf{V}}^\mu \right) \right)^2 \\
\tilde{\mathcal{A}}_5 &= \frac{1}{(4\pi)^2} \left(\text{Tr} \left(\tilde{\mathbf{V}}_\mu \tilde{\mathbf{V}}_\nu \right) \right)^2 \\
\tilde{\mathcal{A}}_6 &= \frac{1}{(4\pi)^2} \text{Tr} \left((\mathcal{D}_\mu \tilde{\mathbf{V}}^\mu)^2 \right),
\end{aligned} \tag{B8}$$

with the EW covariant derivative applied to $\tilde{\mathbf{V}}$ as

$$\mathcal{D}_\mu \tilde{\mathbf{V}}^\mu = \partial_\mu \tilde{\mathbf{V}}^\mu + i g \left[\tilde{\mathbf{W}}_\mu, \tilde{\mathbf{V}}^\mu \right] + i g' \left[\tilde{\mathbf{B}}_\mu, \tilde{\mathbf{V}}^\mu \right]. \tag{B9}$$

The coefficients \tilde{c}_i are expected to be all of the same order of magnitude, according to the effective field theory approach, except for the coefficients of the operators in $\mathcal{L}_{CH}^{(0)}$ which are taken equal to 1, leading to canonical kinetic terms. The expansion is stopped at the first order in inverse powers of the scale associated to the $SO(5)/SO(4)$ breaking, and therefore no 6-derivative operators have been considered into $\Delta\mathcal{L}_{CH}$.

Couplings to gluons are expected to appear due to fermionic loops: however, we are not aware of any explicit computation of such contributions present in the literature and it is beyond the scope of the present paper to perform such computation.

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